

Risk Adjusted Performance Measurement

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Analyzing Portfolio Performance

- Attribution Analysis
 - Difference between manager return and benchmark return broken down into two components:
 - Selection – difference in performance due to selection of individual assets
 - Allocation - difference in performance due to allocation across sectors
- Risk Analysis
 - Difference in manager's performance from benchmark due to risk
 - Beta more or less than benchmark beta of 1
 - Standard deviation more or less than benchmark standard deviation
- These two analyses are typically done independently
- Implicitly assumes manager's portfolio same risk as benchmark when doing attribution analysis
- Previous attempts to combine them done incorrectly (to be discussed)

Sector Attribution – the Basic Math of Brinson-Hood-Beebower (BHB)

formula	Component	Explanation
$\sum W_p R_p - \sum W_b R_b =$	Total return difference	Wtd ave fund return – wtd ave benchmark return
$\sum W_b \times (R_p - R_b)$	Selection effects	Benchmark weight applied to return difference
$+ \sum (W_p - W_b) \times R_b$	Allocation effects	Benchmark return applied to weight difference
$+ \sum (W_p - W_b) \times (R_p - R_b)$	Cross product terms	Difference in weights x difference in returns

Sector Attribution – the Basic Math of Brinson-Fachler (BF)

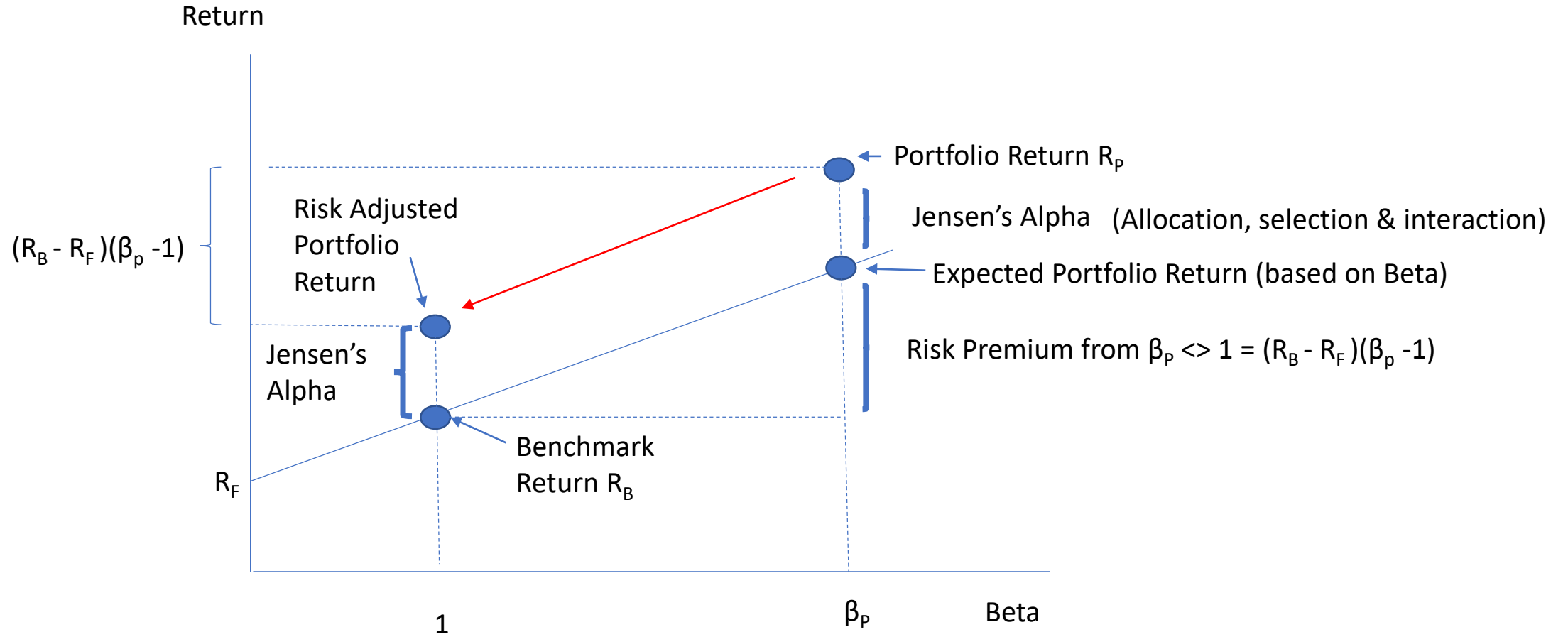
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Overall benchmark return (weighted average of sectors). To have a over allocation score there must be a positive allocation to a sector with an above average return. Or an under allocation to a sector with a below average return.

The basic idea (simplified)

- Risk Adjust the Manager's portfolio return for each sector
- What would the return be if it had the same risk as the benchmark?
 - Same beta of 1
 - OR Same standard deviation
- Use the risk adjusted manager return in traditional attribution analysis
 - Brinson-Hood-Beebower (BHB)
OR
 - Brinson-Fachler (BF)
 - BF has a better interpretation of allocation results for each sector
 - Total allocation impact the same as BHB
 - BF used in example presented below.

Risk Adjusted Portfolio Return



From previous illustration

Overall Benchmark Return

Beta for sector

- Risk Adjusted Return for sector $i = (R_B - R_F)(\beta_{pi} - 1)$
- Use the Risk Adjusted Return in place of the nominal return
- R_B is the overall benchmark return (wtd ave of sectors)
- R_F is the risk free rate
- β_{pi} is the beta for the manager's portfolio sectors

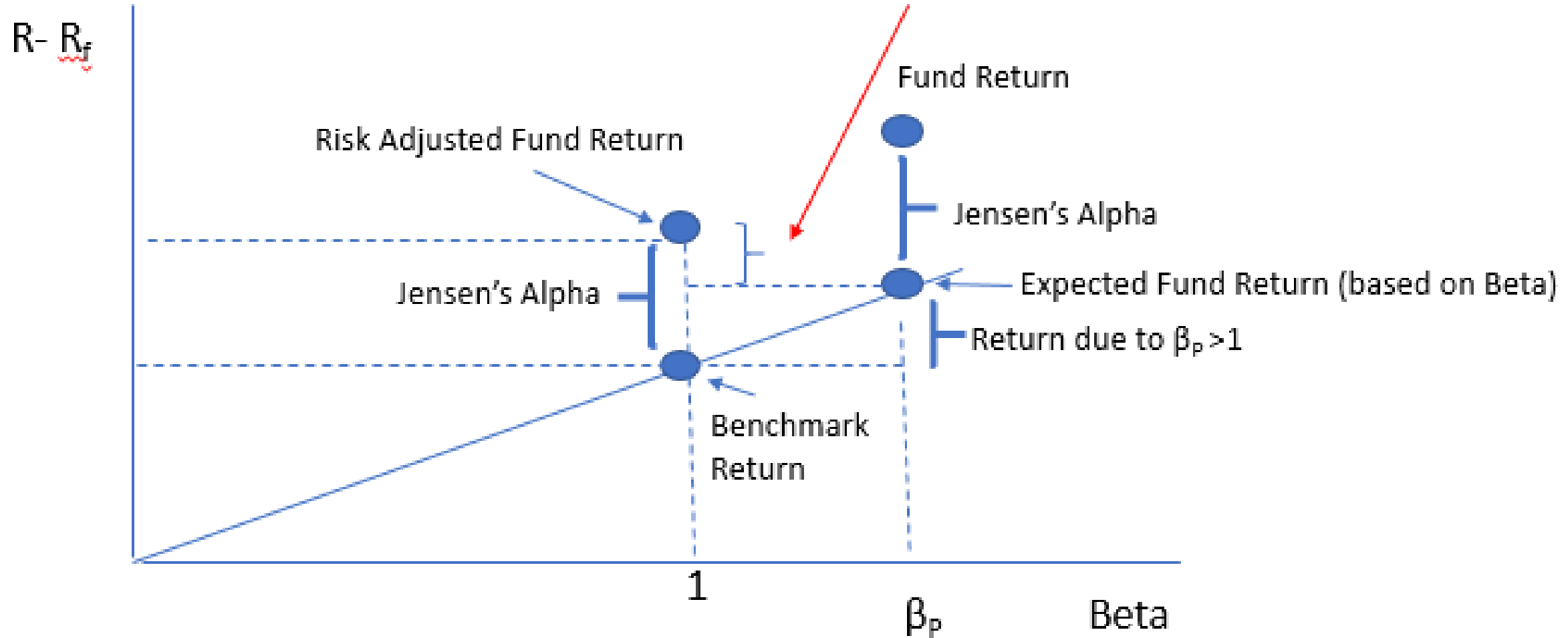
Slight complication

- By definition the benchmark has a beta of 1
- But individual sectors (property types, locations) could have a beta that is $\neq 1$.
- The weighted average of the sector betas has to be 1.
- Therefore we need to also risk adjust each benchmark sector
 - Manager could have allocated more to a riskier sector & vice versa
 - Manager could have selected riskier properties within a sector & vice versa
 - Need an apples to apples comparison (same risk) of the manager's return vs. benchmark return in each sector
- Same formula: $(R_B - R_F)(\beta_{bi} - 1)$
 - But done for each sector using the beta for that sector
 - But R_B is still the overall benchmark return (as the theory suggests)
 - β_{bi} is the beta for the benchmark sector

Previous attempts

- Ankrim (1992) in Journal of Performance Measurement (JOPM) tried to use a CAPM approach but mis-applied the math
 - Removed some of the manager's alpha from the risk adjusted return! (See next slide)
- Menchero (1996/97) in JOPM used an Information Ratio approach, but that doesn't reconcile to a return, let alone Jensen's alpha.
- Obeid (2005) in JOPM modified Ankrim's model, but fell short of reconciling to Jensen's alpha.
- Bacon (2008) in Practical Portfolio Performance Measurement and Attribution uses Fama's concept of net selectivity, but assigns all systematic risk to allocation and does not reconcile to Jensen's alpha
- Spaulding (2016) in JPOM used a similar approach, but used M^2 as the risk adjusted return which does not reconcile with Jensen's alpha.
 - $M^2 = R_f + (R_p - R_f) \times \sigma_B / \sigma_p$ which starts with the manager return and reduces it to have the same standard deviation as the benchmark.
 - But the CAPM prices risk based on the benchmark expected return – not the manager's return.

Ankrim doesn't give the manager credit for this portion of alpha



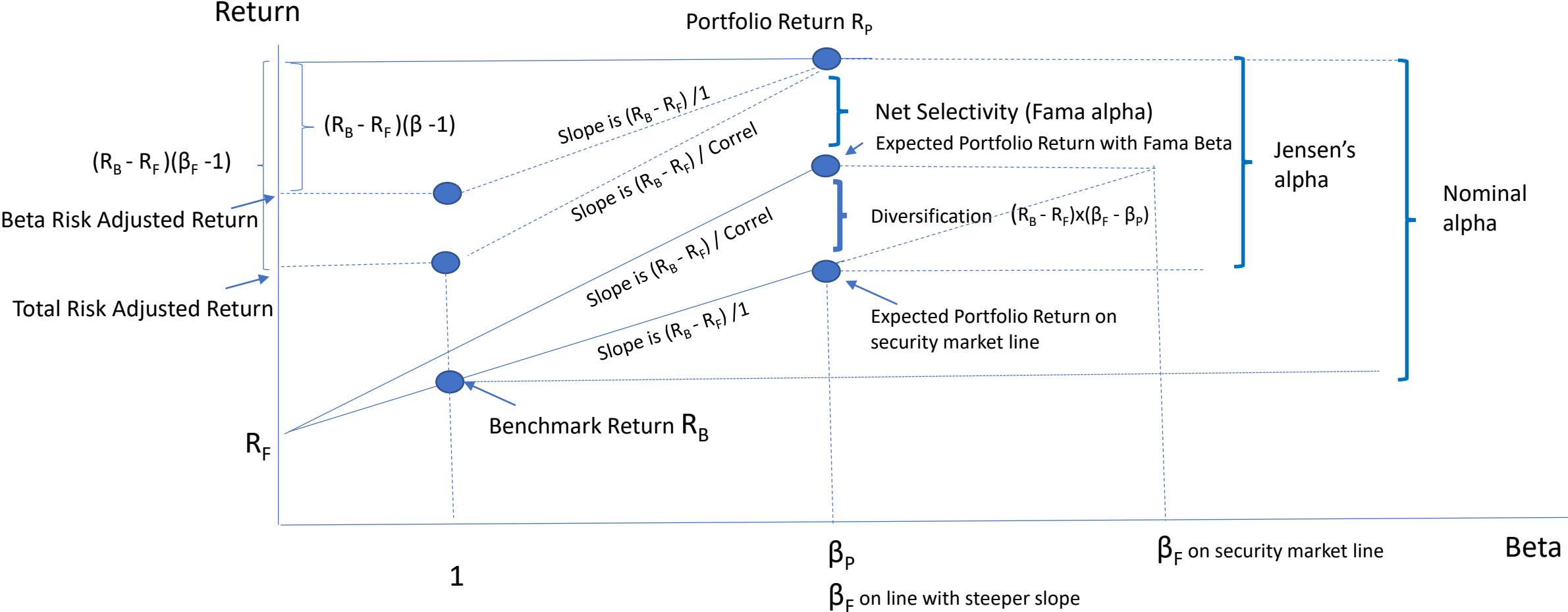
An Extension

- Fama introduced concept of “net selectivity”
- Adjusts for difference in what has been referred to as “Fama beta”
- Fama Beta: $\beta_F = \beta_P / \text{correl}(R_P, R_B)$ OR $\beta_F = \sigma_P / \sigma_B$
- According to Fama this may be more applicable to investors who do not hold well diversified portfolios.
- It captures systematic and unsystematic (“non-diversification”) risk.

$$\text{Expected Return} = R_F + \underbrace{(R_B - R_F) \beta_P}_{\text{Premium for systematic risk}} + \underbrace{(\beta_F - \beta_P) (R_B - R_F)}_{\text{Premium for unsystematic risk}}$$

$$\text{Fama Alpha} = R_P - \left\{ R_F + \underbrace{(R_B - R_F) \beta_P}_{\text{Premium for systematic risk}} + \underbrace{(\beta_F - \beta_P) (R_B - R_F)}_{\text{Premium for unsystematic risk}} \right\}$$

Risk Adjusted Portfolio Return



Using Fama Beta

- Fama Risk-adjusted return = $(R_B - R_F)(\beta_F - 1)$
- Use Fama beta in place of regular beta to risk adjust returns
- Must be done for each sector (portfolio and benchmark)
- Using both regular and Fama beta provides a boundary within which the risk adjustment could be made

Example

- Created a pseudo manager fund by aggregating all separate accounts in the NCREIF database (\$201.2 billion)
- Used properties in the NCREIF ODCE index as benchmark (\$260.3 billion)
 - NFI-ODCE = Open end diversified core equity index
 - Industry benchmark used by core open-end funds since it is a fund level index
- Null Hypothesis: The aggregation of all separate accounts should perform about the same as ODCE.
 - Same managers in general
 - Large portfolio of accounts with core to core plus strategies

Exhibit II: Data Set for Brinson Attribution Analysis

A	B	C	D	E	F	G
2		Portfolio Weights	Benchmark Weights	Portfolio Returns	Benchmark Returns	<i>Nominal Alpha</i>
3	Apartment	23.0%	23.5%	8.9%	7.4%	
4	Hotel	1.4%	1.2%	9.9%	7.9%	
5	Industrial	10.5%	12.8%	13.9%	13.5%	
6	Office	34.7%	40.3%	8.2%	9.1%	
7	Retail	30.4%	22.2%	9.1%	8.8%	
8	Total	100.0%	100.0%	9.3%	9.2%	<i>0.1%</i>

Appears the portfolio just slightly beat the benchmark return.



Before risk adjustment

Exhibit IV: Calculate Attribution Components for Nominal Alpha

A	B	M	N	O	P
17		BF Allocation (H x K)	BF Selection (I x L)	BF Interaction (J)	<i>Nominal Alpha</i> (M+N+O)
18	Apartment	0.0%	0.3%	0.0%	0.3%
19	Hotel	0.0%	0.0%	0.0%	0.0%
20	Industrial	-0.1%	0.0%	0.0%	-0.1%
21	Office	0.0%	-0.4%	0.1%	-0.3%
22	Retail	0.0%	0.1%	0.0%	0.1%
23	Total	-0.1%	0.1%	0.1%	0.1%

On a nominal (before risk adjustment) basis the Manager appears to have performed well in Apartment and poorly in Office. (Under-weighted office which has a slightly below average benchmark return.) Also manager appears to have positive alpha in retail.

Sector returns adjusted for beta risk for both the portfolio and the benchmark.

Exhibit VII: Data Necessary to Calculate Beta Risk-Adjusted Performance Attribution

A	B	C	D	V (port)	V (bench)
		Portfolio Weights	Benchmark Weights	Risk-Adjusted Portfolio Returns	Risk-Adjusted Benchmark Returns
41					
42	Apartment	23.0%	23.5%	9.5%	8.6%
43	Hotel	1.4%	1.2%	7.9%	7.4%
44	Industrial	10.5%	12.8%	19.7%	19.9%
45	Office	34.7%	40.3%	7.4%	8.1%
46	Retail	30.4%	22.2%	2.6%	5.4%
47	Total	100.0%	100.0%	7.7%	9.2%

Nominal returns from previous slide that were not risk adjusted

E	F
Portfolio Returns	Benchmark Returns
8.9%	7.4%
9.9%	7.9%
13.9%	13.5%
8.2%	9.1%
9.1%	8.8%
9.3%	9.2%

On a risk adjusted basis the manager only earned 7.7% vs 9.2% for benchmark

*Benchmark sector returns are different but overall benchmark return is the same
Since by definition the benchmark still has to have a beta of 1.*

Exhibit IX: Calculate Additional Market Risk as Nominal Alpha Less Jensen's Alpha

A	B	P	Z	AA
		Nominal Alpha	Jensen's Alpha	Market Risk (P-Z)
57				
58	Apartment	0.3%	0.2%	0.1%
59	Hotel	0.0%	0.0%	0.0%
60	Industrial	-0.1%	-0.3%	0.2%
61	Office	-0.3%	-0.2%	-0.1%
62	Retail	0.1%	-1.2%	1.2%
63	Total	0.1%	-1.4%	1.5%

Nominal Alpha 0.1%
 Less: market Risk -1.5%
 Jensen's Alpha -1.4%

Should have earned 1.5% more based on additional market risk.

This is what we want to break down between selection and allocation

Where did alpha really come from?

Exhibit VIII: **Beta Risk-Adjusted Attribution Components**

A	B	W	X	Y	Z
49		Risk Adjusted BF Allocation (C-D) x [V (bench) - F]	Risk Adjusted BF Selection [V (port) - V (bench)] x D	Risk Adjusted BF Interaction [C-D] X [V (port) x V (bench)]	Jensen's Alpha (W+X+Y)
50	Apartment	0.0%	0.2%	0.0%	0.2%
51	Hotel	0.0%	0.0%	0.0%	0.0%
52	Industrial	-0.3%	0.0%	0.0%	-0.3%
53	Office	0.1%	-0.3%	0.0%	-0.2%
54	Retail	-0.3%	-0.6%	-0.2%	-1.2%
55	Total	-0.5%	-0.7%	-0.2%	-1.4%

On a risk-adjusted basis, portfolio under-performed the benchmark by 140 basis points.

Retail is the primary reason for under-performance. Poor job in allocation and selection in retail. (Recall on a nominal basis it appeared the manager did okay with retail.

Also poor job in selecting office properties and under weighted industrial which hurt selection since industrial performed well in the benchmark.

Exhibit XIII: Substitute Portfolio Returns with Fama Beta Risk-Adjusted Returns

Beta risk-adjusted returns from previous slide

A	B	C	D	AC	F
		Portfolio Weights	Benchmark Weights	Fama Beta Risk Adjusted Portfolio Returns	Fama Beta Risk Adjusted Benchmark Returns
41					
42	Apartment	23.0%	23.5%	9.6%	8.9%
43	Hotel	1.4%	1.2%	6.7%	-2.9%
44	Industrial	10.5%	12.8%	17.2%	17.4%
45	Office	34.7%	40.3%	6.8%	8.8%
46	Retail	30.4%	22.2%	-3.6%	6.0%
47	Total	100.0%	100.0%	5.4%	9.2%

V (port)	V (bench)
Risk-Adjusted Portfolio Returns	Risk-Adjusted Benchmark Returns
9.5%	8.6%
7.9%	7.4%
19.7%	19.9%
7.4%	8.1%
2.6%	5.4%
7.7%	9.2%

Sector returns adjusted for Fama beta risk for both the portfolio and the benchmark. Benchmark weighted average standard deviation must be used.*

Benchmark sector returns are different but overall benchmark return is the same.

**Actual standard deviation lower if sectors not perfectly correlated but we want to remove any diversification with Fama Beta.*

Exhibit XIV: Calculate **Fama's Alpha** Attribution

A	B	AD	AE	AF	AG
49		Risk Adjusted BF Allocation (C-D) x [V (bench) - F]	Risk Adjusted BF Selection [V (port) - V (bench)] x D	Risk Adjusted BF Interaction	<i>Fama's Alpha</i> (AD+AE+AF)
50	Apartment	0.0%	0.2%	0.0%	0.2%
51	Hotel	0.0%	0.1%	0.0%	0.1%
52	Industrial	-0.2%	0.0%	0.0%	-0.2%
53	Office	0.0%	-0.8%	0.1%	-0.7%
54	Retail	-0.3%	-2.1%	-0.8%	-3.2%
55	Total	-0.5%	-2.7%	-0.6%	-3.8%

*Jensen's
alpha from
previous slide*

Z
Jensen's Alpha
0.2%
0.0%
-0.3%
-0.2%
-1.2%
-1.4%

Manager did even worse
on basis of Fama alpha

Exhibit XV: Calculate the Non-Diversification Risk Premium

A	B	C	AG	AH	AA	AI
57		Nominal Alpha	Fama's Alpha	Total Risk (C-AG)	Market Risk	Non-Diversification Risk (AH-AA)
58	Apartment	0.3%	0.2%	0.2%	0.1%	0.0%
59	Hotel	0.0%	0.1%	-0.1%	0.0%	-0.1%
60	Industrial	-0.1%	-0.2%	0.1%	0.3%	-0.1%
61	Office	-0.3%	-0.7%	0.4%	-0.1%	0.5%
62	Retail	0.1%	-3.2%	3.3%	1.2%	2.0%
63	Total	0.1%	-3.8%	3.9%	= 1.5%	+ 2.4%

To compensate for non-diversification the investor would have needed to earn another 2.4% over the 1.5% for market risk.

Because the manager over-weighted Retail by 8.2% and Retail was the most risky sector, it was assigned 2% of the non-diversification risk premium.

Exhibit XVII: Summary of Risk-Adjusted Performance Attribution

Risk Adjusted Attribution Analysis	Total	Allocation	Selection	Interaction
Nominal Alpha	0.1%	-0.1%	0.1%	0.1%
Market Risk	1.5%	0.4%	0.8%	0.3%
Jensen's Alpha	-1.4%	-0.5%	-0.7%	-0.2%
Non-Diversification Risk	2.4%	-0.1%	2.0%	0.5%
Fama Alpha (Net Selectivity)	-3.8%	-0.5%	-2.7%	-0.6%

Poor selection main cause of worse performance.

Exhibit XVIII: Summary Attribution by Sector

Summary Attribution by Sector						
Risk-Adjusted Attribution Analysis	Apartment	Hotel	Industrial	Office	Retail	Total
Nominal Alpha	0.3%	0.0%	-0.1%	-0.3%	0.1%	0.1%
Jensen's Alpha	0.2%	0.0%	-0.3%	-0.2%	-1.2%	-1.4%
Market Risk Premium (Nominal Alpha - Jensen's Alpha)	0.1%	0.0%	0.3%	-0.1%	1.2%	1.5%
Non-Diversification Risk Premium (Jensen's Alpha - Fama's Alpha)	0.0%	-0.1%	-0.1%	0.5%	2.0%	2.4%
Total Risk Premium	0.2%	-0.1%	0.1%	0.4%	3.3%	3.9%
Fama's Alpha (Nominal Alpha - Total Risk Premium)	0.2%	0.1%	-0.2%	-0.7%	-3.2%	-3.8%

Retail hurt performance the most on a risk adjusted basis. Without considering risk, Office was the worst performer

Exhibit XIX: Detail Attribution by Sector

Detail Attribution Components by Sector						
Risk-Adjusted Attribution Analysis	Apartment	Hotel	Industrial	Office	Retail	Total
Nominal Allocation	0.0%	0.0%	-0.1%	0.0%	0.0%	-0.1%
Nominal Selection	0.3%	0.0%	0.0%	-0.4%	0.1%	0.1%
Nominal Interaction	0.0%	0.0%	0.0%	0.1%	0.0%	0.1%
Total Nominal Alpha	0.3%	0.0%	-0.1%	-0.3%	0.1%	0.1%
Jensen Allocation	0.0%	0.0%	-0.3%	0.1%	-0.3%	-0.6%
Jensen Selection	0.2%	0.0%	0.0%	-0.3%	-0.6%	-0.7%
Jensen Interaction	0.0%	0.0%	0.0%	0.0%	-0.2%	-0.2%
Total Jensen's Alpha	0.2%	0.0%	-0.3%	-0.2%	-1.2%	-1.4%
Fama Allocation	0.0%	0.0%	-0.2%	0.0%	-0.3%	-0.5%
Fama Selection	0.2%	0.1%	0.0%	-0.8%	-2.1%	-2.7%
Fama Interaction	0.0%	0.0%	0.0%	0.1%	-0.8%	-0.6%
Total Fama's Alpha	0.2%	0.1%	-0.2%	-0.7%	-3.2%	-3.8%

Reconciliation to Alpha's

72		Fama Beta	Beta
73	Beta's	1.475	1.188
74	Benchmark Excess Return	8.2%	8.2%
75	Risk Free Rate	1.0%	1.0%
76	Expected Return	13.1%	10.7%
77	Portfolio Actual Return	9.3%	9.3%
78	Alpha	-3.8%	-1.4%
		Fama Alpha	Jensen's Alpha

From previous slide

Risk Adjusted Attribution Analysis	Total
Nominal Alpha	0.1%
Market Risk	1.5%
Jensen's Alpha	-1.4%
Non-Diversification Risk	2.4%
Fama Alpha (Net Selectivity)	-3.8%

We now have a range of risk adjustments depending on how we think the investors should have been compensated for non-diversification.

Conclusion

1. The difference between the portfolio and benchmark return is decomposed into the following components:
 1. Risk premium due to the portfolio beta
 2. Risk premium due to lack of diversification (optional)
 3. Net selection
 4. Net allocation
 5. Interaction
2. The Model
 - neutralizes the differences in sector betas between portfolio and benchmark;
 - preserves manager's alpha when analyzing Brinson attribution components of active management, and
 - incorporates total risk by analyzing systematic and unsystematic risk, an extension of the work of Fama's concept of net selectivity.