

## An Analysis of Default Risk in the Home Equity Conversion Mortgage (HECM) Program

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### Aging in Place: Analyzing the Use of the Reverse Mortgage to Preserve Independent Living & Enable Housing Stability

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#### Research Program (2012-2016)

#### 1. Empirical Modeling

- HECM terminations & default
- Take-up of HECMs
- HECM loan terms and withdrawal behaviors

#### 2. Survey of Counseled Seniors

- Longer term well-being of HECM borrowers
- May 2014, targeting 5,000 respondents: (1) current HECM borrowers, (2) terminated HECM borrowers, and (3) seniors who sought counseling but did not get a reverse mortgage.

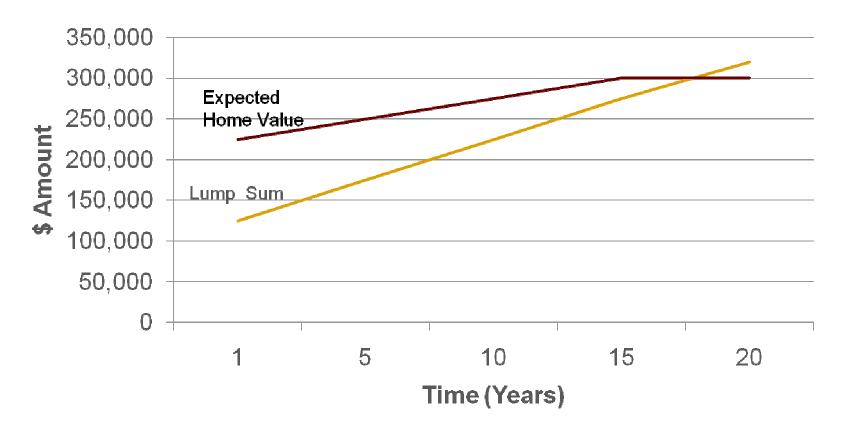
#### 3. Post Origination Monitoring Pilot

- RCT design; financial planning and reminders after closing
- Target date: June 2014

#### **Reverse Mortgage 101**

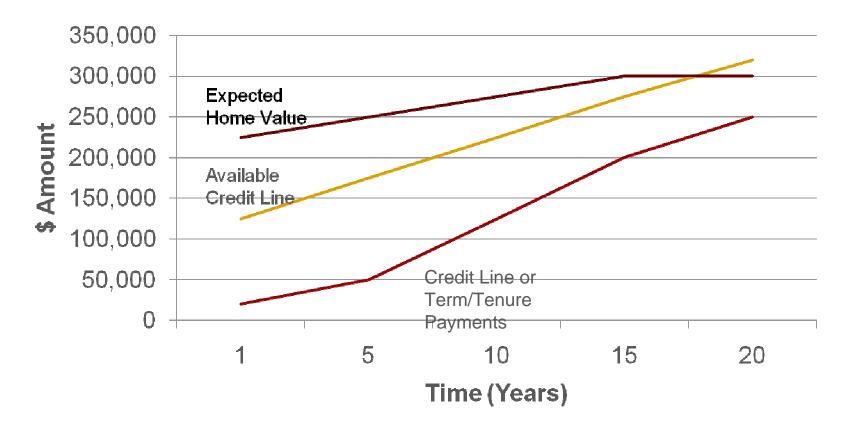
- Extract equity from the home through a mortgage that does not become due until the last borrower sells the home, moves out permanently, or dies, as long as the borrower meets the obligations of the mortgage note
  - Obligations include living in the home as primary residence, pays
     property taxes, homeowners insurance, homeowners association dues
     and assessments, and maintains the home.
- No payments on the loan are required during the life of the loan. Money borrowed, plus associated interest and fees, are added to the balance due that continues to grow over time (mortgage "in reverse")
  - Line of Credit
  - Tenure or Term (similar to annuity)
  - Lump Sum Distribution
  - Some combination of the above

#### **Reverse Mortgage Debt**



Maximum Claim Amount (home value at closing)= \$225,000 Initial Principal Limit = \$125,000

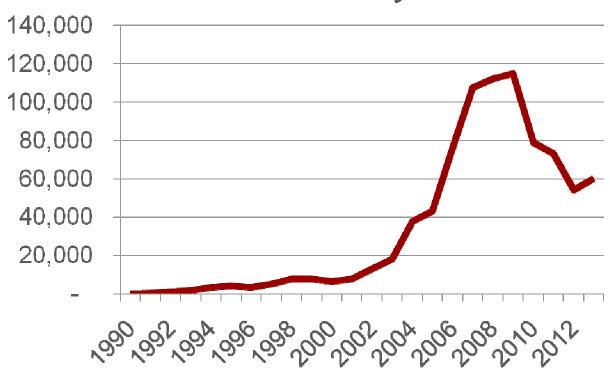
#### **Reverse Mortgage Debt**



Maximum Claim Amount (home value at closing)= \$225,000 Initial Principal Limit = \$125,000

## Take-Up of HECMs

## **Number of Loans by Year**



Source: Author's calculations from HUD HECM data

#### **HECM Tax & Insurance Default**

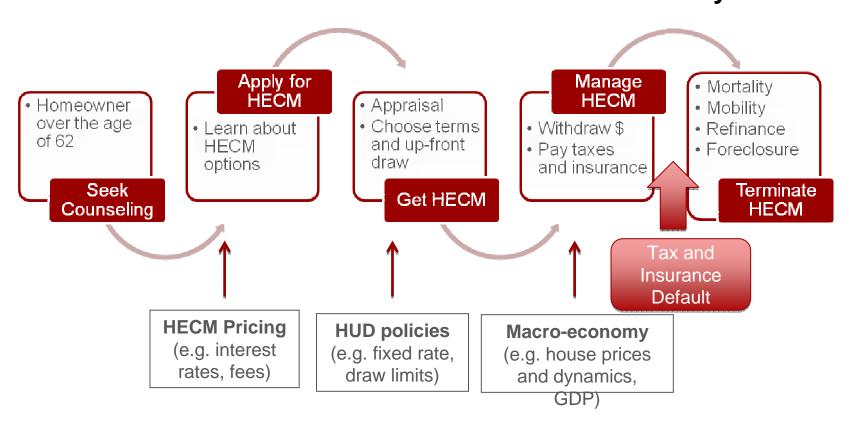
#### Motivation

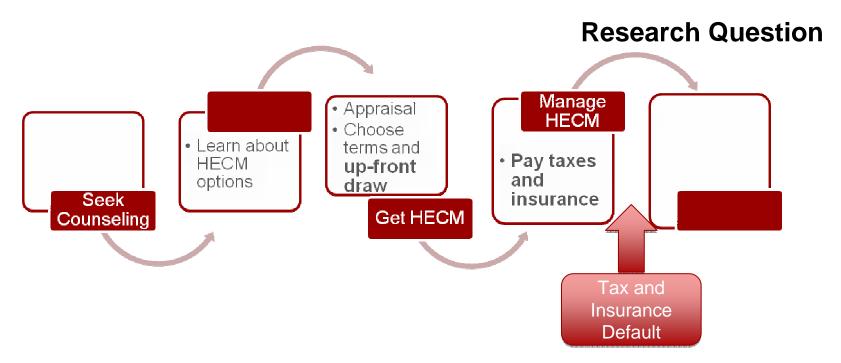
- 9.4 percent of all HECM borrowers in technical default due to nonpayment of property taxes and/or homeowner's insurance, as of February 2012
- HUD policy response:
  - Limits on up-front draw %
  - Financial assessment requirement (underwriting criteria)
  - Life expectancy set-aside (LESA)

#### Explanatory factors at origination expected to be associated with default

- Lack of financial resources or excessive expenditures
  - Income, assets, available credit, debt burdens
- History of poor credit performance
  - Credit score, missed installment/revolving payments, tax liens
- Management of HECM funds
  - Initial withdraw %

## **HECM Lifecyle Decisions**





#### This Paper:

- •What factors at the time of origination are associated with future tax and insurance ddefault of verse mortgage borrowers?
  - Accounting for HECM take-up among counseled households
  - Modeling the endogeneity of the up-front withdrawal %

M15

Rodda, Lam, and Youn (2004) Szymanoski, Enriquez, and DiVenti (2007), Moulton, Stephanie, 3/3/2014

#### **Previous research**

- Potential demand for reverse mortgages
  - General demand: Venti and Wise 1991; Merrill, Finkel, and Kutty 1994;
     Rasmussen, Megbolugbe, and Morgan 1995; Mayer and Simons 1994; Costa-Font, Gil, and Mascarilla 2010
  - Life-cycle model: Nakajima and Telyukova 2013
- Take-Up of reverse mortgages
  - General take-up: Shan 2011
  - House price dynamics: Haurin et al. 2013
  - Selection and moral hazard: Davidoff and Welke 2004; Davidoff 2013; 2014
- Performance of reverse mortgages
  - **Termination outcomes**: Szymanoski, DiVenti, and Chow, 2000; Szymanoski, Enriquez, and DiVenti 2007; Rodda, Lam and Youn 2004; Bishop and Shan 2008
  - Ruthless terminations: Davidoff and Wetzel 2013; Davidoff 2013
  - Pricing risks: Szymanoski 1994; Chinloy and Megbolugbe 1994
  - Tax and insurance default: IFE 2011; 2012; 2013

#### **Data**

- 1. CredAbility counseling data
  - 2006 2011, including more than 30,000 seniors
  - NCOA's Financial Interview Tool (FIT) data after October 2010
- 2. Equifax credit report data
  - time of counseling & annually thereafter
- 3. Economic indicators
  - national, state and county level, time varying
- 4. HUD HECM loan data
  - includes T&I defaults

COUNSELED HECM T&I Default (N=28,129) (N=16,283) (N=1,173)

57.9%

7.2%

## **Sample Data: Demographics**

#### **Demographic Characteristics, Reverse Mortgage Counseling Clients 2006-2011**

	COUNSELED (N=28,129)	HECM (N=16,283)	T&I DEFAULT (N=1,173)
	mean	mean	mean
Hispanic	11.0%	9.4%	18.8%
Race - white	63.3%	68.1%	42.4%
Race - black	16.6%	12.7%	26.3%
Race - Asian	0.9%	0.9%	0.7%
First language - not English	7.2%	5.6%	13.0%
Unmarried Male	16.1%	15.7%	20.5%
Unmarried Female	36.2%	39.3%	42.1%
Age - youngest household member	72	72	71
Education - bachelors degree	11.0%	11.1%	7.5%
Education - high school diploma	32.9%	31.0%	28.3%
Education - advanced degree	4.8%	4.6%	3.0%
Education - some college	19.8%	19.7%	13.1%

Source: CredAbility Counseling Data, 2006-2011

## **Sample Data: Financials**

#### Financial Characteristics, Reverse Mortgage Counseling Clients 2006-2011

		COUNSELED (N=28,129)		HECM (N=16,283)		efault 1,173)
	mean	median	mean	median	mean	median
Monthly income - sum, non-missing	2,311	1,880	2,337	1,918	1,849	1,534
Taxes - property taxes/income, non-missing	0.091	0.063	0.096	0.067	0.112	0.082
Revolving account high credit - balance	20,672	5,092	23,231	7,979	7,077	83
Revolving balance/income	0.231	0.031	0.252	0.038	0.161	0.000
Installment balance/income	0.236	0	0.221	0	0.291	0

Source: CredAbility Counseling Data, 2006-2011

## **Sample Data: Borrower Risk**

#### **Borrower Risk Characteristics, Reverse Mortgage Counseling Clients 2007-2011**

	FULL DATA (N=28,129)		HECM (N=16,283)			
	mean	median	mean	median	mean	median
FICO score, non-missing (N=26,253)	678	690	693	709	597	593
Mortgage- foreclosure started	0.019	0	0.010	0	0.028	0
Bankruptcy - any in last 12 months	0.010	0	0.006	0.000	0.011	0
Tax lien - percent with a tax lien or judgment	0.102		0.079		0.169	
Mortgage past due, 2+ months	0.057	0	0.035	0	0.101	0

Source: CredAbility Counseling Data, 2007-2011



#### **Sample Data: Property & Mortgage Characteristics**

**Property & Mortgage Characteristics, Reverse Mortgage Counseling Clients 2006-2011** 

	COUNSELED (N=28,129)				T&I Defau 3) (N=1,173)	
	mean	median	mean	median	mean	median
Monthly mortgage payments	498	87	462	0	487	0
HELOC indicator	0.130		0.140		0.093	
Excess home value amount	18,006	0	17,220	0	12,696	0
HECM-Estimated Net IPL (Take-Up Model)	84,555	61,087	93,186	70,763	71,997	55,395
HECM-Actual Net IPL (Default Model)			83,147	62,603	63,851	46,823
HECM- Actual IPL (Withdrawal Model)			139,977	115,688	129,410	109,662
HECM- Home debt/IPL (Withdrawal Model)			0.387	0.352	0.470	0.521
Up-front draw % (Default Model)			0.771	0.939	0.883	0.934
Exposure- # days since origination as of July 1, 2012			799	<i>675</i>	1,118	1,140
Fixed rate policy indicator	0.781		0.736		0.477	

Source: CredAbility & HUD data, 2006-2011



A household's selection into HECM is modeled as

$$HECM_{i} = \begin{cases} 1 & \text{if } X_{i}\beta_{1} + S_{i}\gamma + u_{1i} > 0 \\ 0 & \text{otherwise} \end{cases}$$

D<sub>i</sub>=1 indicates that borrower i defaults. D<sub>i</sub> is observed only if the person is a HECM borrower: HECM<sub>i</sub>=1.

$$D_{i} = \begin{cases} 1 \text{ if } X_{i}\beta_{2} + Z_{i}\delta + W_{i}\alpha + u_{2i} > 0 \text{ and } HECM_{i} = 1\\ 0 \text{ if } X_{i}\beta_{2} + Z_{i}\delta + W_{i}\alpha + u_{2i} \leq 0 \text{ and } HECM_{i} = 1 \end{cases}$$

A household's initial withdrawal W<sub>i</sub> is modeled as

$$W_i = X_i \beta_3 + H_i \theta + u_{3i}$$

W<sub>i</sub> is observed only if the household obtained HECM. We estimate the three equations simultaneously (selection, withdrawal and T&I default)



S <sub>i</sub> only in selection equation	Z <sub>i</sub> only in default equation	H <sub>i</sub> only in withdrawal equation
Estimated Net Initial Principal Limit	Net Initial Principal Limit	Actual Initial Principal Limit
Excess of home value above MCA	% upfront draw (W <sub>i</sub> )	Mortgage/IPL
State house price deviation from the state's long run norm	Difference between the date of origination and July 2012	Fixed rate policy dummy (=1 after Apr 1, 2009) Interaction between fixed rate dummy & spread between average interest rates of FRM and ARM.

 $X_i$  in all equations, includes demographic characteristics, income, property tax burden, debt burdens, FICO, credit characteristics, delinquencies, state and year fixed effects

# Truncated Bivariate Probit, HECM Take-Up Conditional on up-front draw%, Select Significant Results (p<.05)

	Marginal
	Effects
Race- black	-0.0661
Unmarried male	0.0528
Unmarried female	0.1175
Age youngest owner	0.0200
Age youngest owner squared	-0.0001
Non-English Speaking	-0.0517
Monthly Income	0.0122
Revolving balance/income	0.0540
Mortgage payments	-0.0163
HELOC indicator	0.0175
FICO score	0.0005
Foreclosure started	-0.0701
Bankruptcy in past 12 months	-0.0996
Mortgage past due 2+ months	-0.0461
Tax lien or judgment	-0.0352
Estimated IPL net	0.0005
Excess home value	-0.0003

Fixed effects for state and year.

**Results: HECM Take-Up** 



#### MLE, Up-Front Withdrawal %, Accounting for Partial Observability of HECM; Select Significant Results (p<.05)

	b
Hispanic	0.0238
Race - black	0.0387
Unmarried male	0.0339
Unmarried female	0.0147
Non-English speaking	0.0556
Education- college	-0.0292
Education- post graduate	-0.0324
Monthly income	0.0061
Property taxes/income	-0.2176
Revolving balance/income	0.0416
Installment balance/income	0.0137
Available revolving credit	-0.0003
FICO score	-0.0002
Mortgage past due 2+ months	-0.0306
IPL_postHECM	-0.0001
Home Debt/Actual IPL	0.3266
Policy dummy for fixed rate	

Fixed effects for state and year. Other model variables not significant at p<.05

#### **Results: % Withdrawal**

- Minority borrowers have higher initial draws than non-minority borrowers.
- Borrowers who completed education at a four year college or graduate school have about 3% lower initial draws.
- While tax liens are associated with slightly higher initial draws, a higher property tax burden is associated with taking less money out up front.
- Higher revolving and installment debt is associated with slightly higher initial draws.
- A 100 point increase in credit score is associated with a 2% decrease in the initial draw.
- A \$10,000 increase in available credit is associated with a 3% decrease in the initial draw.
- The fixed rate policy beginning in 2009 is associated with a 5.5% increase in the initial draw.



## Truncated Bivariate Probit, T&I Default Conditional on HECM and up-front draw%, Select Significant Results (p<.05)

	Marginal Effects
Hispanic	0.0257
Race - black	0.0197
Unmarried male	0.0269
Monthly income	-0.0058
Property taxes/income	0.0546
Revolving balance/income	-0.0152
Available revolving credit	-0.0005
FICO score	-0.0004
Mortgage past due 2 months+	0.0224
Tax lien or judgment	0.0205
Up-front draw %	0.1251

Fixed effects for state and year, and controls for exposure days. Other model variables not significant at p<.05

#### **Results: T&I Default**

- Minority borrowers' default rates are about 2 percentage points higher than non-minority borrowers.
- A \$1,000 increase in monthly income is associated with about a ½ (.58) percentage point decrease in default rate.
- An increase in property tax to income burden is associated with increased default.
- An additional \$10,000 in available credit is associated with a ½ (.5) percentage point decrease in the default rate.
- A 100 point increase in credit score is associated with a 4 percentage point decrease in the default rate.
- Borrowers in default on their mortgage, or with tax liens or judgments have default rates that are about 2 percentage points higher.
- A 10 percentage point increase in upfront draw % is associated with a 1.2 percentage point increase in default rate.

### **Policy Simulations**

- Impose new up-front draw limits
  - No mortgage debt: 60% IPL
  - If mortgage debt: payoff, up-front costs + 10% IPL
  - Simulation assumptions:
    - All still get HECMs, take lesser of observed draw or max draw limit
- Impose credit risk thresholds & LESA affordability
  - Apply thresholds based on credit score and credit report attributes
  - If hhld fails threshold, see if hhld could afford LESA from net IPL
    - Fail, afford LESA: get HECM, T&I default = 0
    - Fail, not afford LESA: do not get HECM (T&I default not observed)
  - Simulation assumptions:
    - LESA estimates based on 2008-2010 property tax rates
    - Threshold is hard cut-off requiring LESA
    - Those who are required to take LESA have IPL reduced by LESA \$
    - T&I default rate for those taking LESA is 0%

## **Policy Simulations: Initial Withdrawal Limits**

#### **Predicted Default Probability Conditional on HECM**

	%∆ in	Full Sample Predicted	Full Sample Predicted T&I Default		o/ A :
	Total HECM	T&I Default Rate Before	Rate After Initial Draw	∆ in T&I Default	% Δ in T&I Default
	volume	Policy	Limit <sup>1</sup>	Rate <sup>2</sup>	Rate <sup>3</sup>
HECM Sample		7.20%	5.55%	-1.65%	-22.88%
LESA Based on Credit Score Thresholds					
Observations with credit scores		7.03%	5.43%	-1.60%	-22.77%
LESA for Credit score less than 500					
LESA for Credit score less than 580					
LESA Based on Credit Thresholds					
Observations with credit reports					
LESA for Delinquent Mortgage/In Foreclosure					
LESA for Tax Lien					
LESA for Delinquent Installment					
LESA for Delinquent Revolving					
LESA for Any Above					

## **Policy Simulations: Credit Score Thresholds**

#### **Predicted Default Probability Conditional on HECM**

	%∆ in Total HECM volume	Full Sample Predicted T&I Default Rate Before Policy	Full Sample Predicted T&I Default Rate After Initial Draw Limit <sup>1</sup>	Δ in T&I Default Rate <sup>2</sup>	% Δ in T&I Default Rate <sup>3</sup>
HECM Sample		7.20%	5.55%	-1.65%	22.88%
LESA Based on Credit Score Thresholds					
Observations with credit scores		7.03%	5.43%	-1.60%	-22.77%
LESA for Credit score less than 500	-1.07%		4.55%	-2.48%	-35.30%
LESA for Credit score less than 580	-4.45%		2.90%	-4.13%	-58.75%
LESA Based on Credit Thresholds					
Observations with credit reports					
LESA for Delinquent Mortgage/In Foreclosure					
LESA for Tax Lien					
LESA for Delinquent Installment					
LESA for Delinquent Revolving					
LESA for Any Above					

## **Policy Simulations: Credit Score Thresholds**

#### **Predicted Default Probability Conditional on HECM**

	%∆ in	Full Sample Predicted	Full Sample Predicted T&I Default	A : TO I	0/ A in TQ1
	Total HECM	T&I Default Rate Refore	Rate After Initial Draw	Δ in T&I Default	% Δ in T&I  Default
	volume	Policy	Limit <sup>1</sup>	Rate <sup>2</sup>	Rate <sup>3</sup>
HECM Sample		7.20%	5.55%	-1.65%	22.88%
LESA Based on Credit Score Thresholds					
Observations with credit scores		7.03%	5.43%	-1.60%	-22.77%
LESA for Credit score less than 500	-1.07%		4.55%	-2.48%	-35.30%
LESA for Credit score less than 580	-4.45%		2.90%	-4.13%	-58.75%
LESA Based on Credit Thresholds					
Observations with credit reports		6.76%	5.18%	-1.58%	-23.43%
LESA for Delinquent Mortgage/In Foreclosure	-2.09%		4.26%	-2.5%	-37.03%
LESA for Tax Lien	-2.26%		4.22%	-2.54%	-37.64%
LESA for Delinquent Installment	-0.66%		4.89%	-1.87%	-27.63%
LESA for Delinquent Revolving	-2.71%		3.94%	-2.82%	-41.71%
LESA for Any Above	-5.64%		2.74%	-4.02%	-59.46%

#### **Policy Implications & Conclusions**

- Policy viability of HECM program
  - T&I defaults that result in foreclosure can contribute to fiscal insolvency of the MMI fund
  - "Headline risk" of program and perceived public value
- Mitigating default risk while not (overly) restricting access
  - Restrictions on initial withdrawals
  - Credit risk thresholds & LESA affordability
- Next steps:
  - Generalizing empirical model
  - Other outcomes of consumer well-being
  - Post-origination monitoring as innovation to reduce default

Questions?

## **Hypotheses**

Variable	Withdrawal %	Default
Financial Resources & Expenditure	s	
Income	-	-
Borrowing capacity	-	-
Property tax burden	?	+
Debt ratios	+	+
Borrower Credit Risk		
Credit score	-	-
Tax liens	+	+
Missed mortgage payments	?	+
Management of HECM Funds		
Initial withdrawal %		+
Net IPL		-
Home debt/IPL	+	
Fixed rate, full draw policy	+	

#### HECM Selection

$$y_{i1}^* = x_{i1}'\beta_1 + z_{i1}'\alpha_1 + \epsilon_{i1} \tag{1}$$

The household selects HECM ( $y_{i1} = 1$ ) if  $y_{i1}^* > 0$ . Among HECM borrowers, initial withdrawal as a percentage of initial principal limit ( $w_i$ ) and whether the borrower has defaulted on tax or insurance ( $y_{i2}$ ) are observed.

T&I default

$$y_{i2}^* = x_{i2}'\beta_2 + z_i'\alpha_2 + w_i\gamma_2 + \epsilon_{i2}$$
(2)

The household defaults  $(y_{i2} = 1)$  if  $y_{i2}^* > 0$  and  $y_{i1} = 1$ .

Withdrawal<sup>1</sup>

$$w_i = x_{i3}'\beta_3 + z_i'\alpha_3 + \epsilon_{i3} \tag{3}$$

In Eqs. (1)-(3),  $z_i$  are common variables,  $x_{i1}$ ,  $x_{i2}$ ,  $x_{i3}$  are unique to each equation respectively. The unobservables  $\begin{bmatrix} \epsilon_{i1} & \epsilon_{i2} & \epsilon_{i3} \end{bmatrix}$  are jointly normal with mean 0 and variance

$$\Sigma = \begin{bmatrix} 1 & \rho_{12} & \rho_{13}\sigma \\ \rho_{12} & 1 & \rho_{23}\sigma \\ \rho_{13}\sigma & \rho_{23}\sigma & \sigma^2 \end{bmatrix}$$
(4)

The unobservables are assumed to be independent from  $x_{i1}, x_{i2}, x_{i3}$  and  $z_i$ . The withdrawal  $w_i$  is correlating with  $\epsilon_{i2}$  if  $\epsilon_{i3}$  is correlated with  $\epsilon_{i2}$ .

There are 3 cases.

	case 1	case 2	case 3	
$y_{i1}$	1	1	0	HECM take-up
$y_{i2}$	1	0		t&i default
$w_i$	observed	observed		initial withdrawal

• Case 1: the household selects HECM  $y_{i1} = 1$ , withdraws  $w_i$ , and defaults  $y_{i2} = 1$ . The joint density is

$$l_{i1}(\theta) = f(y_{i1} = 1, y_{i2} = 1, w_i = w | x_{i1}, x_{i2}, x_{i3}, z_i)$$

$$= \int_{-x'_{i1}\beta_1 - z'_{i}\alpha_1} \int_{-x'_{i2}\beta_2 - z'_{i}\alpha_2 - w_{i}\gamma} \phi_3(\epsilon_1, \epsilon_2, w - x'_{i3}\beta_3 - z'_{i}\alpha_3) d\epsilon_1 d\epsilon_2$$

$$= \int_{-x'_{i1}\beta_1 - z'_{i}\alpha_1} \int_{-x'_{i2}\beta_2 - z'_{i}\alpha_2 - w_{i}\gamma} \phi_{\epsilon_1, \epsilon_2 | \epsilon_3 = w - x'_{i3}\beta_3 - z'_{i}\alpha_3}(\epsilon_1, \epsilon_2) f(w | x_{i3}, z_i) d\epsilon_1 d\epsilon_2$$

$$= f(w | x_{i3}, z_i) \int_{-x'_{i1}\beta_1 - z'_{i}\alpha_1} \int_{-x'_{i2}\beta_2 - z'_{i}\alpha_2 - w_{i}\gamma} \phi_{\epsilon_1, \epsilon_2 | \epsilon_3 = w - x'_{i3}\beta_3 - z'_{i}\alpha_3}(\epsilon_1, \epsilon_2) d\epsilon_1 d\epsilon_2$$

$$= f(w | x_{i3}, z_i) P(y_{i1} = 1, y_{i2} = 1 | x_{i1}, x_{i2}, x_{i3}, z_i, w_i = w)$$

$$(7)$$

In Eq. (5),  $\phi_3$  is the density of trivariate normal distribution with mean  $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$  and variance  $\Sigma$  as in Eq. (4). Then in Eq. (6), the trivariate normal density is written as a product of the marginal density of  $\epsilon_{i3}$  and the conditional density of  $\epsilon_{i1}$ ,  $\epsilon_{i2}$  on  $\epsilon_{i3} = w - x'_{i3}\beta_3 - z'_i\alpha_3$ . The terms in Eq. (7) are

$$\log f(w_i = w | x_{i3}, z_i) \propto -\frac{1}{2} \log \sigma^2 - \frac{1}{2\sigma^2} (w - x'_{i3}\beta_3 - z'_i\alpha_3)^2,$$

$$\log P(y_{i1} = 1, y_{i2} = 1 | x_{i1}, x_{i2}, x_{i3}, z_i, w_i = w) = \log \Phi_2(x'_{i1}\beta_1 + z'_{i}\alpha_1, x'_{i2}\beta_2 + z'_{i}\alpha_2 + w_i\gamma_2; \bar{\mu}_{i,1}, \bar{\Sigma}_1)$$

where  $\Phi_2(\cdot,\cdot;\bar{\mu}_{i,1},\Sigma_1)$  is the cdf of a bivariate normal  $(\bar{\mu}_{i,1},\Sigma_1)$ . Using the property of multivariate normal distribution,

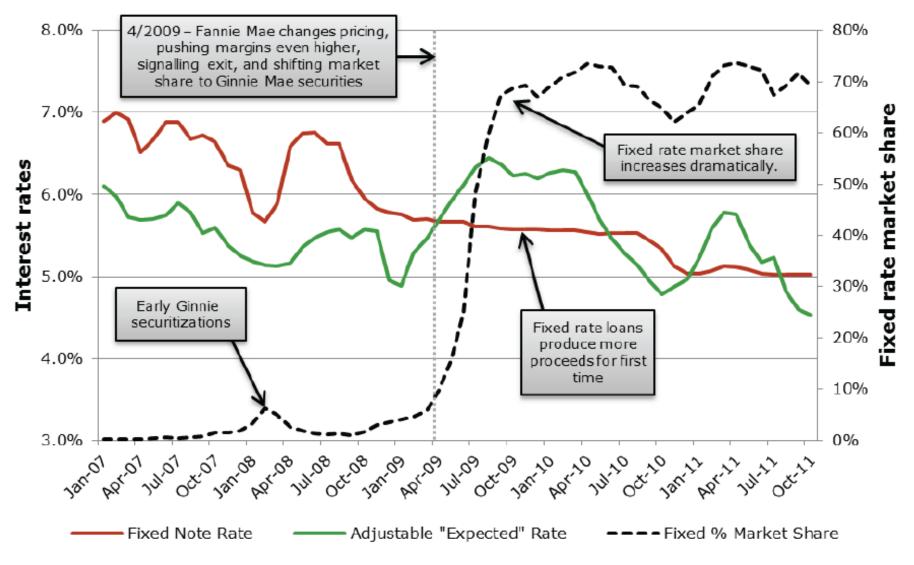
$$\begin{split} \bar{\mu}_{i,1} &= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \Sigma_{12} \Sigma_{22}^{-1} (w_i - x_{i3}' \beta_3 - z_i' \alpha_3) = \begin{pmatrix} -\frac{\rho_{13}}{\sigma} \\ -\frac{\rho_{23}}{\sigma} \end{pmatrix} (w_i - x_{i3}' \beta_3 - z_i' \alpha_3), \\ \bar{\Sigma}_1 &= \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} = \begin{pmatrix} 1 - \rho_{13}^2 & \rho_{12} - \rho_{13} \rho_{23} \\ \rho_{12} - \rho_{13} \rho_{23} & 1 - \rho_{23}^2 \end{pmatrix}, \\ \text{where } \Sigma_{11} &= \begin{pmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{pmatrix}, \, \Sigma_{22} = \sigma^2, \, \Sigma_{12} = \begin{pmatrix} \rho_{13} \sigma \\ \rho_{23} \sigma \end{pmatrix}. \end{split}$$

The full likelihood function is

$$\log L_n(\theta) = \sum_{i=1}^n \{ I(y_{i1} = 1, y_{i2} = 1) \log l_{i1}(\theta)$$
  
+  $I(y_{i1} = 1, y_{i2} = 0) \log l_{i2}(\theta) + I(y_{i1} = 0) \log l_{i3}(\theta) \}$ 

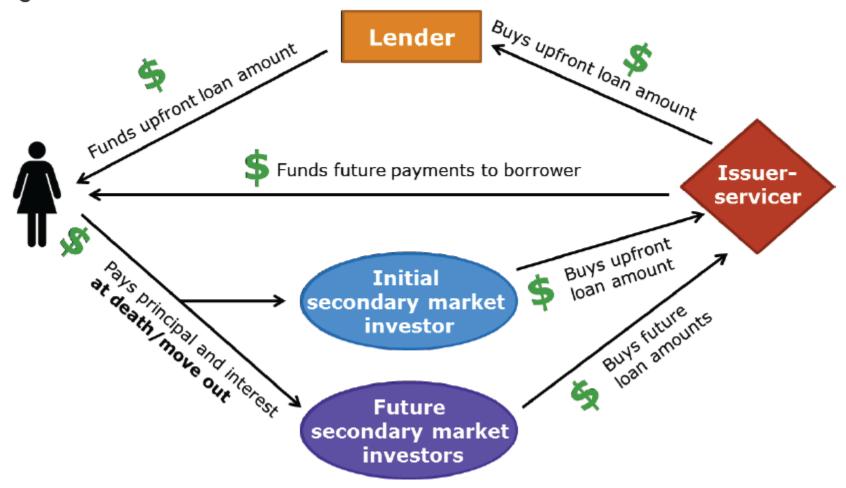
In the maximum likelihood estimation,  $\rho_{12}$ ,  $\rho_{13}$ ,  $\rho_{23}$  and  $\sigma$  are not directly estimated. Directly estimated is a transformation of these parameters,  $\log \sigma$  for  $\sigma$  and  $\mathrm{atanh}\rho = \frac{1}{2}\log\left(\frac{1+\rho}{1-\rho}\right)$  for  $\rho$ . We have  $\rho = \frac{-1+\exp(2\mathrm{atanh}\rho)}{1+\exp(2\mathrm{atanh}\rho)}$ . The parameter space of the transformed variable is unrestricted. The same transformation is used by the Stata routine "heckman".

Figure 38: Interest rates and fixed-rate market share

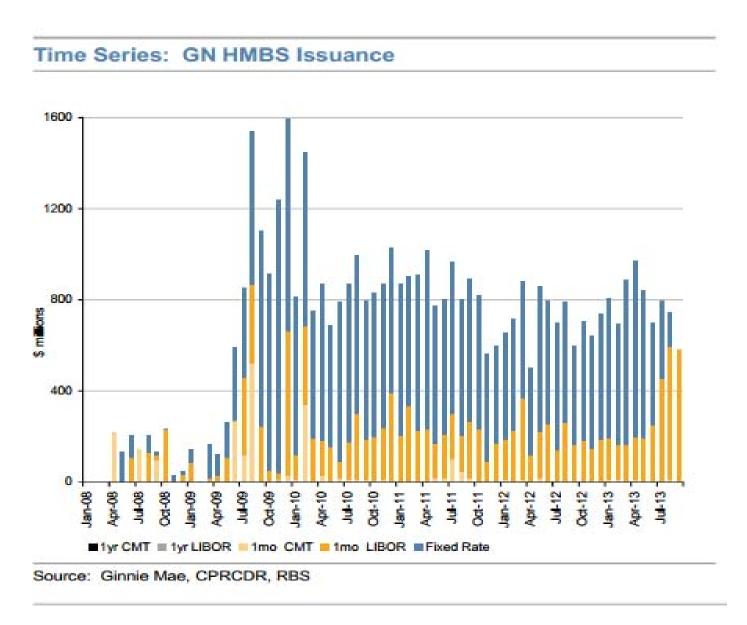


Source: CFPB 2012

Figure 36: Ginnie Mae securitization model



Source: CFPB 2012



http://www.ginniemae.gov/doing\_business\_with\_ginniemae/issuer\_resources/ginnienet/NewIssuerTrainingPresentations/20131122\_HMBS\_Panel.pdf