Stochastic Taxation and Pricing of CMBS REITs

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May 11, 2014
SUMMARY OF FINDINGS

- Major Innovation: Introduction of Stochastic Taxation
- After-tax Risk Premium resolves a substantial part of the Equity Premium Puzzle
- Coefficient of Relative Risk Aversion: 7.43 – 10.59
PRESENTATION STRATEGY

• Review CCAPM
• Outline the Equity Risk Premium Puzzle
• Introducing Stochastic Taxation into the Analysis
• Determining the Coefficient of Risk Aversion
SUMMARY OF CCAPM

Theorem (Lucas Tree-Model (1978)): Assume

- Preferences:
  \[ U_i(c_i) = u_i(c_{it}) + E\left[ \sum_{T=1}^{\infty} b_i^T u_i(c_{it+T}) \right] \quad \forall i \in I. \]
  \[ u_i'(\cdot) > 0, \quad u_i''(\cdot) < 0 \quad \forall i \in I. \]

- Budget Constraint:
  \[
  \sum_{k=1}^{n} z_{ikt+T}(p_{kt+T} + d_{kt+T}) = c_{it+T} + \sum_{k=1}^{n} z_{ikt+T+1} p_{kt+T} \quad \forall i \in I, \\
  \forall T = 0, \ldots, \infty.
  \]

- Supply of Assets:
  \[
  \sum_{i \in I} z_{ikt+T} = \bar{z}_{kt+T} > 0 \quad \forall k = 1, \ldots, n \quad \forall T = 0, \ldots, \infty.
  \]
Then

- Pricing Equation:

$$p_{kt} = E \left[ \frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})} (p_{kt+1} + d_{kt+1}) \right] \forall k = 1, \ldots, n.$$ 

- Efficient Market Hypothesis:

$$p_{kt} = E \left[ \sum_{T=1}^{\infty} \frac{b_i^T u_i'(c_{it+T})}{u_i'(c_{it})} d_{kt+T} \right] \forall k = 1, \ldots, n.$$ 

- Euler Equation:

$$E \left[ \frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})} R_{kt+1} \right] = 1 \forall k = 1, \ldots, n,$$

$$E \left[ \frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})} \right] R_f = 1.$$
COROLLARY 1: Assume
- Lucas (1978) CCAPM
- \( u'_i(c_{it+1}) = \lambda_i \cdot R_{mt+1} \)

Then
- CAPM:

\[
E [R_{kt+1} - R_f] = \beta_k \cdot E [R_{mt+1} - R_f] \quad \forall k = 1, \ldots, n.
\]

COROLLARY 2: Assume
- Lucas (1978) CCAPM
- Identical Agents

Then
- Efficient Market Hypothesis:

\[
p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T \frac{u'(\sum_{k=1}^{n} d_{kt+T})}{u'\left(\sum_{k=1}^{n} d_{kt}\right)} d_{kt+T} \right].
\]
COROLLARY 3: Assume

- Lucas (1978) CCAPM
- Identical Agents
- CRRA: \( u(c) = \frac{c^{1-a}}{1-a} \)

Then

- Efficient Market Hypothesis:

\[
p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T \left( \frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}} \right)^{-a} d_{kt+T} \right].
\]
COROLLARY 4: Assume

- Lucas (1978) CCAPM
- Identical Agents
- CRRA: \( u(c) = \frac{c^{1-a}}{1-a} \)
- \( \ln(c_{t+T}) \sim N(\mu_c, \sigma_c) \ \forall \ T = 1, \ldots, \infty \)
- \( n = 1 \)

Then

\[
p_{kt} = E \left[ \sum_{T=1}^{\infty} b^T \left( \frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}} \right)^{1-a} \right] \cdot d_{kt},
\]

\[
\frac{d_{kt+T}}{p_{kt+T}} = \frac{c_{t+T}}{p_{kt+T}} = \text{constant} \ \forall \ T = 1, \ldots, \infty.
\]
THEOREM (RUBINSTEIN (1976)): Assume
- Lucas (1978) CCAPM
- Identical Agents
- CRRA: \( u(c) = \frac{c^{1-a}}{1-a} \)
- \( \ln(c_{t+T}) \sim N(\mu_c, \sigma_c) \ \forall T = 1, \ldots, \infty \)
- \( \ln(R_{kt+T}) \sim N(\mu_k, \sigma_k) \ \forall T = 1, \ldots, \infty \)
- \( \rho \ln(R_{kt+T}), \ln(c_{t+T}) \geq 0 \ \forall T = 1, \ldots, \infty \)

Then

\[
\ln E[R_{kt+1}] - \ln R_f = a \cdot \text{cov}[\ln R_{kt+1}, \ln(\frac{c_{t+1}}{c_t})].
\]

and

- Black-Scholes-Rubinstein Formula:

\[
\text{Call}(p_{kt}, S, T, \sigma_k, \overline{D}, r_f) = \frac{1}{(1+\overline{D})^T} p_{kt} N(Z_{ks} + \sqrt{T} \sigma_k) - \frac{S}{(1+r_f)^T} N(Z_{ks}),
\]

\[
Z_{ks} = \frac{\ln \frac{p_{kt}}{S} + \ln \frac{1}{(1+\overline{D})^T} + \ln R_f^T}{\sqrt{T} \sigma_k} - \frac{1}{2} \sqrt{T} \sigma_k.
\]
The coefficient of relative risk aversion:

\[ rr(c) = \left[ -\frac{u''(c)c}{u'(c)} \right]. \]

**LEMMA:** Assume

- \( u(c) = \frac{c^{1-a}}{1-a} \)
- \( u'(c) = c^{-a} \)
- \( u''(c) = -a \cdot c^{-a-1} \)
- \( rr(c) = \left[ -\frac{-a \cdot c^{-a-1} \cdot c}{c^{-a}} \right] = a \)

**Equity Premium Puzzle for \( \beta = 1 \) Portfolio, (Mehra and Prescott (1985) and Mehra (2003)):**

\[ a = \frac{\ln(E[R_{mt+1}]) - \ln(R_f)}{\text{COV}[\ln(R_{mt+1}), \ln\left(\frac{C_{t+1}}{C_t}\right)]} = \frac{0.07 - 0.01}{0.00125} = 47.6. \]
CALCULATING TAX YIELD FOR S&P 500

Components of tax yield:
- Dividend tax
- Short-term capital gains tax
- Long-term capital gains tax

Tax yield for the S&P 500 (Sialm (2008)):

\[
TY_{t+1} = \frac{\tau_{d,mt+1} d_{mt+1} + \tau_{SCG,mt+1} SCG_{mt+1} + \tau_{LCG,mt+1} LCG_{mt+1}}{p_{mt}} =
\]

\[
= \tau_{d,mt+1} \cdot \frac{d_{mt+1}}{p_{mt}} + \tau_{SCG,mt+1} \cdot \frac{SCG_{mt+1}}{p_{mt}} + \tau_{LCG,mt+1} \cdot \frac{LCG_{mt+1}}{p_{mt}} =
\]

\[
= \tau_{d,mt+1} \cdot 0.045 + \tau_{SCG,mt+1} \cdot 0.001 + \tau_{LCG,mt+1} \cdot 0.018,
\]

where

- \( p_{mt} \) is the price per share of the market portfolio of risky assets,
- \( d_{mt} \) is the dividend paid per share of the market portfolio of risky assets,
- \( R_{mt+1} = 1 + r_{mt+1} \) is the gross rate of return on the market portfolio of risky assets,
- \( \tau_{d,t+1} \) is the dividend tax,
\[ \tau_{t+1}^{SCG} \] is the tax on short-term capital gains, \\
\[ \tau_{t+1}^{LCG} \] is the tax on long-term capital gains, \\
\[ SCG_{t+1} \] are realized short-term capital gains, \\
\[ LCG_{t+1} \] are realized long-term capital gains, and \\
\[ TY_{t+1} \] is the tax yield.

- The dividend yield for the market portfolio of risky assets:
  \[ \frac{d_{mt+1}}{p_{mt}} = 0.045 \]

- The realized short-term capital gains yield for the market portfolio of risky assets:
  \[ \frac{SCG_{mt+1}}{p_{mt}} = 0.001 \]

- The realized long-term capital gains yield for the market portfolio of risky assets:
  \[ \frac{LCG_{mt+1}}{p_{mt}} = 0.018. \]

- Tax yield for the S&P 500 (Sialm (2008)):
  \[ TY_{t+1} = \tau_{mt+1}^{d} \cdot 0.045 + \tau_{t+1}^{SCG} \cdot 0.001 + \tau_{t+1}^{LCG} \cdot 0.018. \]
The tax $\tau_{t+1}$ imposed on the wealth of the S&P 500 stockholders (Magin(2014)):

$$\tau_{t+1} = \frac{\tau^d_{t+1} d_{mt+1} + \tau^{SCG}_{t+1} SCG_{t+1} + \tau^{LCG}_{t+1} LCG_{t+1}}{p_{mt+1} + d_{mt+1}}$$

$$= \frac{\tau^d_{t+1} d_{mt+1}}{p_{mt}} + \frac{\tau^{SCG}_{t+1} SCG_{t+1}}{TY_{t+1}} + \frac{\tau^{LCG}_{t+1} LCG_{t+1}}{TY_{t+1}}.$$

Estimate for the tax $\tau_{t+1}$ imposed on the wealth of the S&P 500 stockholders for 1913-2007:

$$\tau_{t+1} = \left( \tau^d_{t+1} \cdot 0.045 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018 \right) \cdot \frac{1}{R_{mt+1}}.$$
CALCULATING TAX YIELD FOR CMBS REITs

- About 20% of all stock shares are held in taxable accounts.
- Stock dividends are on average taxed at the ordinary income tax rate of about 20%.
- The average effective dividend tax rate estimate:

\[ \tau^d_{t+1} = 0.2 \cdot 0.2 = 0.04. \]

- REITs distribute at least 90% of taxable income to shareholders in the form of dividends.
- REITs dividend distributions constitute a significant portion of the overall before-tax return from REITs.
- REITs dividends are ostensibly taxed as ordinary income.
• Expect that the typical investor in REITs may be subject to below average ordinary income tax rates.

• Many tax exempt institutional investors may be attracted to REITs.

• The average dividend tax rate appropriate for the S&P, in general, may not be appropriate for REITs investors.

\[ \tau_{cmbs	ext{ reits } t+1} = \frac{1}{2} \cdot 0.2 \cdot 0.2 = 0.02. \]

• The average dividend yield for CMBS REITs is more than twice that of the average dividend yield for S&P 500 stocks: 0.123 vs. 0.45.
### TABLE 1: TAX YIELD PARAMETERS

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P 500</th>
<th>Equity REITs</th>
<th>CMBS REITs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{d_{kt+1}}{p_{kt}}$</td>
<td>0.045</td>
<td>0.080</td>
<td>0.123</td>
</tr>
<tr>
<td>$\frac{SCG_{kt+1}}{p_{kt}}$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>$\frac{LCG_{kt+1}}{p_{kt}}$</td>
<td>0.018</td>
<td>0.018</td>
<td>0.018</td>
</tr>
<tr>
<td>$\tau^d_{t+1}$</td>
<td>$\tau^d_{mt+1}$</td>
<td>$[0.25 \cdot \tau^d_{mt+1}, \tau^d_{mt+1}]$</td>
<td>$[0.25 \cdot \tau^d_{mt+1}, \tau^d_{mt+1}]$</td>
</tr>
<tr>
<td>$\tau^{SCG}_{t+1}$</td>
<td>$\tau^{SCG}_{mt+1}$</td>
<td>$\tau^{SCG}_{mt+1}$</td>
<td>$\tau^{SCG}_{mt+1}$</td>
</tr>
<tr>
<td>$\tau^{LCG}_{t+1}$</td>
<td>$\tau^{LCG}_{mt+1}$</td>
<td>$\tau^{LCG}_{mt+1}$</td>
<td>$\tau^{LCG}_{mt+1}$</td>
</tr>
</tbody>
</table>
• Tax Yield for CMBS REITs:

\[
TY_{\text{cmbs reits } t+1} = \frac{\tau^d_{\text{cmbs reits } t+1} d_{\text{cmbs reits } t+1} + \tau^{SCG}_{t+1} SCG_{\text{cmbs reits } t+1} + \tau^{LCG}_{t+1} LCG_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}}
\]

\[
= \tau^d_{\text{cmbs reits } t+1} \cdot \frac{d_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} + \tau^{SCG}_{t+1} \cdot \frac{SCG_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} + \tau^{LCG}_{t+1} \cdot \frac{LCG_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} = 0.02 \cdot 0.123 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018.
\]

• The dividend yield for CMBS REITs:

\[
\frac{d_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} = 0.123
\]

• The realized short-term capital gains yield for CMBS REITs:

\[
\frac{SCG_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} = 0.001
\]

• The realized long-term capital gains yield for CMBS REITs:

\[
\frac{LCG_{\text{cmbs reits } t+1}}{p_{\text{cmbs reits } t}} = 0.018
\]
• Tax Yield for CMBS REITs:

\[ TY_{cmbs \text{ reits } t+1} = 0.02 \cdot 0.123 + \tau_{t+1}^{SCG} \cdot 0.001 + \tau_{t+1}^{LCG} \cdot 0.018. \]

• The mean tax yield for shareholders of CMBS REITs:

\[ E \left[ TY_{cmbs \text{ reits } t+1} \right] = 0.0061. \]
ESTIMATING EXPECTED AFTER-TAX RISK PREMIUMS AND THE COEFFICIENT OF RELATIVE RISK AVERSION FOR CMBS REITs INVESTORS

- Traditional CCAPM Rubinstein (1976) and Lucas (1978) without insecure property rights:

\[
a = \frac{\ln(E[R_{cmbs\ reits\ t+1}]) - \ln(R_f)}{\text{COV} \left[ \ln(R_{cmbs\ reits\ t+1}), \ln\left(\frac{C_{t+1}}{C_t}\right) \right]} = \frac{0.7 \cdot 0.06}{0.00125} = 33.6000.
\]

- Fama and French (2002) dividend growth model:

\[
a = \frac{\ln(E[R_{cmbs\ reits\ t+1}]) - \ln(R_f)}{\text{COV} \left[ \ln(R_{cmbs\ reits\ t+1}), \ln\left(\frac{C_{t+1}}{C_t}\right) \right]} = \frac{0.7 \cdot 0.0255}{0.0178} = 14.2400.
\]
Applying Magin (2014) CCAPM with stochastic taxes $\tau_{cmb\text{s reits } t+1}$:

$$a = \frac{\ln(E[R_{cmb\text{s reits } t+1}]) - \ln(R_{f}) + \ln(E[1-\tau_{cmb\text{s reits } t+1}]) + \text{COV} \left[ \ln(R_{cmb\text{s reits } t+1}), \ln(1-\tau_{cmb\text{s reits } t+1}) \right]}{\text{COV} \left[ \ln(R_{cmb\text{s reits } t+1}), \ln\left( \frac{C_{t+1}}{C_{t}} \right) \right] + \text{COV} \left[ \ln(1-\tau_{cmb\text{s reits } t+1}), \ln\left( \frac{C_{t+1}}{C_{t}} \right) \right]}$$

$$= \frac{0.70.0255-0.0061+0.0002}{0.00125+0.0000} = 9.5354.$$
<table>
<thead>
<tr>
<th>Effective Dividend Tax</th>
<th>Expected Tax Yield</th>
<th>After-tax Risk Premium</th>
<th>Coefficient of Relative Risk Aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.0087</td>
<td>0.0091</td>
<td>7.4273</td>
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<tr>
<td>0.03</td>
<td>0.0074</td>
<td>0.0104</td>
<td>8.4803</td>
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<tr>
<td>0.02</td>
<td>0.0061</td>
<td>0.0117</td>
<td>9.5334</td>
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<td>0.01</td>
<td>0.0048</td>
<td>0.0130</td>
<td>10.5865</td>
</tr>
</tbody>
</table>
# Table 2: Coefficients of Relative Risk Aversion for Different Asset Classes

<table>
<thead>
<tr>
<th>Asset Class</th>
<th>Dividend Yield, %</th>
<th>Coefficient of Relative Risk Aversion</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500 Index Portfolio</td>
<td>4.50</td>
<td>3.76</td>
<td>Magin (2014)</td>
</tr>
<tr>
<td>Equity REITs</td>
<td>8.00</td>
<td>4.32-6.29</td>
<td>Edelstein and Magin (2013)</td>
</tr>
<tr>
<td>CMBS REITs</td>
<td>12.29</td>
<td>7.43-10.59</td>
<td>This Paper</td>
</tr>
</tbody>
</table>