Stochastic Taxation and Pricing of CMBS REITs

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SUMMARY OF FINDINGS

- Major Innovation: Introduction of Stochastic Taxation
- After-tax Risk Premium resolves a substantial part of the Equity Premium Puzzle
- Coefficient of Relative Risk Aversion: 7.43 – 10.59

PRESENTATION STRATEGY

- Review CCAPM
- Outline the Equity Risk Premium Puzzle
- Introducing Stochastic Taxation into the Analysis
- Determining the Coefficient of Risk Aversion

SUMMARY OF CCAPM

Theorem (Lucas Tree-Model (1978)): Assume

- Preferences:

$$U_i(c_i) = u_i(c_{it}) + E[\sum_{T=1}^{\infty} b_i^T u_i(c_{it+T})] \; \forall i \in I.$$
$$u_i'(\cdot) > 0, \; u_i''(\cdot) < 0 \; \forall i \in I.$$

- Budget Constraint:

$$\sum_{k=1}^{n} z_{ikt+T}(p_{kt+T} + d_{kt+T}) = c_{it+T} + \sum_{k=1}^{n} z_{ikt+T+1} p_{kt+T} \; \forall i \in I,$$
$$\forall T = 0, ..., \infty.$$

- Supply of Assets:

$$\sum_{i \in I} z_{ikt+T} = \bar{z}_{kt+T} > 0 \; \forall k = 1, ..., n \; \forall T = 0, ..., \infty.$$

Then

- Pricing Equation:

$$p_{kt} = E\left[\frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})}(p_{kt+1} + d_{kt+1})\right] \; \forall k = 1, ..., n.$$

- Efficient Market Hypothesis:

$$p_{kt} = E\left[\sum_{T=1}^{\infty} \frac{b_i^T u_i'(c_{it+T})}{u_i'(c_{it})} d_{kt+T}\right] \; \forall k = 1, ..., n.$$

- Euler Equation:

$$E\left[\frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})} R_{kt+1}\right] = 1 \; \forall k = 1, ..., n,$$
$$E\left[\frac{b_i u_i'(c_{it+1})}{u_i'(c_{it})}\right] R_f = 1.$$

COROLLARY 1: Assume

- Lucas (1978) CCAPM
- $u_i'(c_{it+1}) = \lambda_i \cdot R_{mt+1}$

Then

- CAPM:

$$E\left[R_{kt+1} - R_f\right] = \beta_k \cdot E\left[R_{mt+1} - R_f\right] \; \forall k = 1, ..., n.$$

COROLLARY 2: Assume

- Lucas (1978) CCAPM
- Identical Agents

Then

- Efficient Market Hypothesis:

$$p_{kt} = E\left[\sum_{T=1}^{\infty} \frac{b^T u'(\sum_{k=1}^{n} d_{kt+T})}{u'(\sum_{k=1}^{n} d_{kt})} d_{kt+T}\right].$$

COROLLARY 3: Assume

- Lucas (1978) CCAPM
- Identical Agents
- CRRA: $u(c) = \frac{c^{1-a}}{1-a}$

Then

- Efficient Market Hypothesis:

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b^T \left(\frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}}\right)^{-a} d_{kt+T}\right].$$

COROLLARY 4: Assume

- Lucas (1978) CCAPM
- Identical Agents
- CRRA: $u(c) = \frac{c^{1-a}}{1-a}$
- $\ln(c_{t+T}) \sim N(\mu_c, \sigma_c)\ \forall T = 1, ..., \infty$
- $n = 1$

Then

$$p_{kt} = E\left[\sum_{T=1}^{\infty} b^T \left(\frac{\sum_{k=1}^{n} d_{kt+T}}{\sum_{k=1}^{n} d_{kt}}\right)^{1-a}\right] \cdot d_{kt},$$

$$\frac{d_{kt+T}}{p_{kt+T}} = \frac{c_{t+T}}{p_{kt+T}} = constant\ \forall T = 1, ..., \infty.$$

THEOREM (RUBINSTEIN (1976)): Assume

- Lucas (1978) CCAPM
- Identical Agents
- CRRA: $u(c) = \frac{c^{1-a}}{1-a}$
- $\ln(c_{t+T}) \sim N(\mu_c, \sigma_c) \; \forall T = 1, ..., \infty$
- $\ln(R_{kt+T}) \sim N(\mu_k, \sigma_k) \; \forall T = 1, ..., \infty$
- $\rho_{\ln(R_{kt+T}),\ \ln(c_{t+T})} \geqslant 0 \; \forall T = 1, ..., \infty$

Then

$$\ln E[R_{kt+1}] - \ln R_f = a \cdot cov[\ln R_{kt+1}, \ln(\frac{c_{t+1}}{c_t})].$$

and

- Black-Scholes-Rubinstein Formula:

$$Call(p_{kt}, S, T, \sigma_k, \overline{D}, rf) = \frac{1}{\left(1+\overline{D}\right)^T} p_{kt} N(Z_{ks} + \sqrt{T}\sigma_k) - \frac{S}{(1+rf)^T} N(Z_{ks}),$$

$$Z_{ks} = \frac{\ln \frac{P_{kt}}{S} + \ln \frac{1}{(1+\overline{D})^T} + \ln R_f^T}{\sqrt{T}\sigma_k} - \frac{1}{2}\sqrt{T}\sigma_k.$$

EQUITY PREMIUM PUZZLE

- The coefficient of relative risk aversion:

$$rr(c) = \left[-\frac{u''(c)c}{u'(c)} \right].$$

LEMMA: Assume
- $u(c) = \frac{c^{1-a}}{1-a}$

Then
- $u'(c) = c^{-a}$
- $u''(c) = -a \cdot c^{-a-1}$
- $rr(c) = \left[-\frac{-a \cdot c^{-a-1} \cdot c}{c^{-a}} \right] = a$
- Equity Premium Puzzle for $\beta = 1$ Portfolio, (Mehra and Prescott (1985) and Mehra (2003)):

$$a = \frac{\ln(E[R_{mt+1}]) - \ln(R_f)}{COV\left[\ln(R_{mt+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right]} = \frac{0.07 - 0.01}{0.00125} = 47.6.$$

CALCULATING TAX YIELD FOR S&P 500

Components of tax yield:

- Dividend tax
- Short-term capital gains tax
- Long-term capital gains tax

Tax yield for the S&P 500 (Sialm (2008)):

$$TY_{t+1} = \frac{\tau^d_{t+1} d_{mt+1} + \tau^{SCG}_{t+1} SCG_{mt+1} + \tau^{LCG}_{t+1} LCG_{mt+1}}{p_{mt}} =$$

$$= \tau^d_{mt+1} \cdot \frac{d_{mt+1}}{p_{mt}} + \tau^{SCG}_{t+1} \cdot \frac{SCG_{mt+1}}{p_{mt}} + \tau^{LCG}_{t+1} \cdot \frac{LCG_{mt+1}}{p_{mt}} =$$

$$= \tau^d_{mt+1} \cdot 0.045 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018,$$

where

p_{mt} is the price per share of the market portfolio of risky assets,
d_{mt} is the dividend paid per share of the market portfolio of risky assets,
$R_{mt+1} = 1 + r_{mt+1}$ is the gross rate of return on the market portfolio of risky assets,
τ^d_{t+1} is the dividend tax,

τ_{t+1}^{SCG} is the tax on short-term capital gains,
τ_{t+1}^{LCG} is the tax on long-term capital gains,
SCG_{t+1} are realized short-term capital gains,
LCG_{t+1} are realized long-term capital gains, and
TY_{t+1} is the tax yield.

- The dividend yield for the market portfolio of risky assets:

$$\frac{d_{mt+1}}{p_{mt}} = 0.045$$

- The realized short-term capital gains yield for the market portfolio of risky assets:

$$\frac{SCG_{mt+1}}{p_{mt}} = 0.001$$

- The realized long-term capital gains yield for the market portfolio of risky assets:

$$\frac{LCG_{mt+1}}{p_{mt}} = 0.018.$$

- Tax yield for the S&P 500 (Sialm (2008)):

$$TY_{t+1} = \tau_{mt+1}^{d} \cdot 0.045 + \tau_{t+1}^{SCG} \cdot 0.001 + \tau_{t+1}^{LCG} \cdot 0.018.$$

• The tax τ_{t+1} imposed on the wealth of the S&P 500 stockholders (Magin(2014)):

$$\tau_{t+1} = \frac{\tau^{d}_{t+1} d_{mt+1} + \tau^{SCG}_{t+1} SCG_{t+1} + \tau^{LCG}_{t+1} LCG_{t+1}}{p_{mt+1} + d_{mt+1}} =$$

$$= \underbrace{\frac{\tau^{d}_{t+1} d_{mt+1} + \tau^{SCG}_{t+1} SCG_{t+1} + \tau^{LCG}_{t+1} LCG_{t+1}}{p_{mt}}}_{Tax\ Yield,\ TY_{t+1}} \cdot \underbrace{\frac{p_{mt}}{p_{mt+1} + d_{mt+1}}}_{1/R_{mt+1}} = \frac{TY_{t+1}}{R_{mt+1}},$$

• Estimate for the tax τ_{t+1} imposed on the wealth of the S&P 500 stockholders for 1913-2007:

$$\tau_{t+1} = \underbrace{\left(\tau^{d}_{t+1} \cdot 0.045 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018\right)}_{Tax\ Yield,\ TY_{t+1}} \cdot \frac{1}{R_{mt+1}}.$$

CALCULATING TAX YIELD FOR CMBS REITs

• About 20% of all stock shares are held in taxable accounts.
• Stock dividends are on average taxed at the ordinary income tax rate of about 20%.
• The average effective dividend tax rate estimate:

$$\tau^{d}_{t+1} = 0.2 \cdot 0.2 = 0.04.$$

• REITs distribute at least 90% of taxable income to shareholders in the form of dividends.
• REITs dividend distributions constitute a significant portion of the overall before-tax return from REITs.
• REITs dividends are ostensibly taxed as ordinary income.

• Expect that the typical investor in REITs may be subject to below average ordinary income tax rates.

• Many tax exempt institutional investors may be attracted to REITs.

• The average dividend tax rate appropriate for the S&P, in general, may not be appropriate for REITs investors.

• The average effective dividend tax rate estimate:

$$\tau^{d}_{cmbs\ reits\ t+1} = \frac{1}{2} \cdot 0.2 \cdot 0.2 = 0.02.$$

• The average dividend yield for CMBS REITs is more than twice that of the average dividend yield for S&P 500 stocks: 0.123 vs. 0.45.

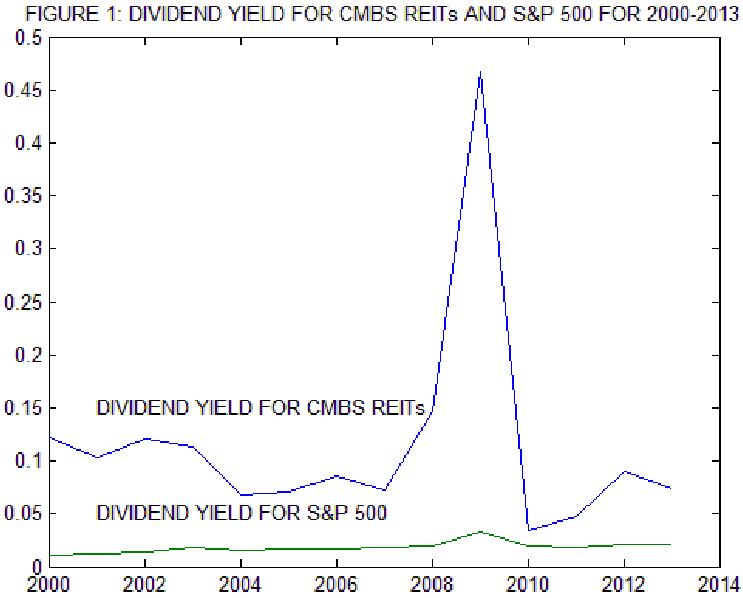
FIGURE 1: DIVIDEND YIELD FOR CMBS REITs AND S&P 500 FOR 2000-2013
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2000
2002
2004
2006
2008
2010
2012
2014
DIVIDEND YIELD FOR CMBS REITs
DIVIDEND YIELD FOR S&P 500

TABLE 1: TAX YIELD PARAMETERS

	S&P 500	Equity REITs	CMBS REITs
$\frac{d_{kt+1}}{p_{kt}}$	0.045	0.080	0.123
$\frac{SCG_{kt+1}}{p_{kt}}$	0.001	0.001	0.001
$\frac{LCG_{kt+1}}{p_{kt}}$	0.018	0.018	0.018
τ^{d}_{t+1}	τ^{d}_{mt+1}	$\left[0.25 \cdot \tau^{d}_{mt+1}, \ \tau^{d}_{mt+1}\right]$	$\left[0.25 \cdot \tau^{d}_{mt+1}, \ \tau^{d}_{mt+1}\right]$
τ^{SCG}_{t+1}	τ^{SCG}_{mt+1}	τ^{SCG}_{mt+1}	τ^{SCG}_{mt+1}
τ^{LCG}_{t+1}	τ^{LCG}_{mt+1}	τ^{LCG}_{mt+1}	τ^{LCG}_{mt+1}

- Tax Yield for CMBS REITs:

$$TY_{cmbs\ reits\ t+1} =$$

$$= \frac{\tau^{d}_{cmbs\ reits\ t+1} d_{cmbs\ reits\ t+1} + \tau^{SCG}_{t+1} SCG_{cmbs\ reits\ t+1} + \tau^{LCG}_{t+1} LCG_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} =$$

$$=$$

$$\tau^{d}_{cmbs\ reits\ t+1} \cdot \frac{d_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} + \tau^{SCG}_{t+1} \cdot \frac{SCG_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} + \tau^{LCG}_{t+1} \cdot \frac{LCG_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} =$$

$$= 0.02 \cdot 0.123 + \tau^{SCG}_{t+1} \cdot 0.001 + \tau^{LCG}_{t+1} \cdot 0.018.$$

- The dividend yield for CMBS REITs:

$$\frac{d_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} = 0.123$$

- The realized short-term capital gains yield for CMBS REITs:

$$\frac{SCG_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} = 0.001$$

- The realized long-term capital gains yield for CMBS REITs:

$$\frac{LCG_{cmbs\ reits\ t+1}}{p_{cmbs\ reits\ t}} = 0.018$$

- Tax Yield for CMBS REITs:

$$TY_{cmbs\ reits\ t+1} = 0.02 \cdot 0.123 + \tau_{t+1}^{SCG} \cdot 0.001 + \tau_{t+1}^{LCG} \cdot 0.018.$$

- The mean tax yield for shareholders of CMBS REITs:

$$E\left[TY_{cmbs\ reits\ t+1}\right] = 0.0061.$$

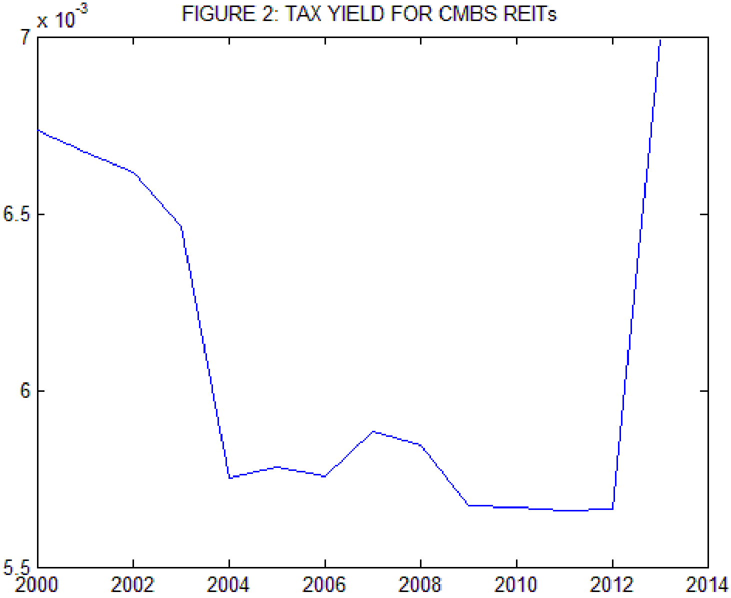
FIGURE 2: TAX YIELD FOR CMBS REITs
x 10^-3
7
6.5
6
5.5
2000
2002
2004
2006
2008
2010
2012
2014

ESTIMATING EXPECTED AFTER-TAX RISK PREMIUMS AND THE COEFFICIENT OF RELATIVE RISK AVERSION FOR CMBS REITs INVESTORS

- Traditional CCAPM Rubinstein (1976) and Lucas (1978) without insecure property rights:

$$a = \frac{\ln(E[R_{cmbs\ reits\ t+1}]) - \ln(R_f)}{COV\left[\ln(R_{\ cmbs\ reits\ t+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right]} = \frac{0.7 \cdot 0.06}{0.00125} = 33.6000.$$

- Fama and French (2002) dividend growth model:

$$\overbrace{E\left[R_{cmbs\ reits\ t+1}\right] - R_f}^{0.7 \cdot 0.0255} = \overbrace{\beta_{cmbs\ reits}}^{0.7}\left(\overbrace{E\left[R_{mt+1}\right] - R_f}^{0.0255}\right) = 0.0178.$$

$$a = \frac{\overbrace{\ln\left(E\left[R_{cmbs\ reits\ t+1}\right]\right) - \ln(R_f)}^{0.7 \cdot 0.0255}}{\underbrace{COV\left[\ln(R_{cmbs\ reits\ t+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right]}} = \frac{0.0178}{0.00125} = 14.2400.$$

- Applying Magin (2014) CCAPM with stochastic taxes $\tau_{cmbs\ reits\ t+1}$:

$$a = \frac{\ln(E[R_{cmbs\ reits\ t+1}]) - \ln(R_f) + \ln(E[1-\tau_{cmbs\ reits\ t+1}]) + COV[\ln(R_{cmbs\ reits\ t+1}),\ \ln(1-\tau_{cmbs\ reits\ t+1})]}{COV\left[\ln(R_{cmbs\ reits\ t+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right] + COV\left[\ln(1-\tau_{cmbs\ reits\ t+1}),\ \ln\left(\frac{C_{t+1}}{C_t}\right)\right]}$$

$$= \frac{0.7 \cdot 0.0255 - 0.0061 + 0.0002}{0.00125 + 0.0000} = 9.5354.$$

TABLE 1: NUMERICAL SIMULATIONS

Effective Dividend Tax	Expected Tax Yield	After-tax Risk Premium	Coefficient of Relative Risk Aversion
0.04	0.0087	0.0091	7.4273
0.03	0.0074	0.0104	8.4803
0.02	0.0061	0.0117	9.5334
0.01	0.0048	0.0130	10.5865

TABLE 2: COEFFICIENTS OF RELATIVE RISK AVERSION FOR DIFFERENT ASSET CLASSES

Asset Class	Dividend Yield, %	Coefficient of Relative Risk Aversion	Source
S&P 500 Index Portfolio	4.50	3.76	Magin (2014)
Equity REITs	8.00	4.32-6.29	Edelstein and Magin (2013)
CMBS REITs	12.29	7.43-10.59	This Paper